

Length Weight Fiber Distribution

One of many ways to characterize fiber mats is by the average fiber diameter distribution. There are many ways to define the average fiber but most appropriate for filter media is the length weighted average fiber size and fiber distribution. Other average fiber diameters are used, such as a number distribution but the number distribution does not account for fibers of different lengths.

The length weighted fiber diameter is defined as the sum of the fiber diameters times length divided by the sum of the lengths

$$\bar{d} = \frac{1}{L_{Total}} \sum L_i d_i \quad (1)$$

This average fiber diameter represents the mean fiber size of a distribution of fiber diameters. To determine the frequency distribution you can use images of the fiber mats to measure diameters and lengths. The log normal frequency distribution fits the data better than normal frequency distribution for some applications such as electrospun nanofibers [1].

The normal frequency distribution is given by

$$f_i = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(d_i - \bar{d})^2}{2\sigma^2}\right) \quad (2)$$

where σ is the standard deviation

$$\sigma = \sqrt{\frac{\sum L_i (d_i - \bar{d})^2}{L_{Total}}} \quad (3)$$

The log normal frequency distribution is defined a bit different as

$$f_i = \frac{1}{d_i \ln \sigma_g \sqrt{2\pi}} \exp\left(-\frac{(\ln d_i - \ln \bar{d}_g)^2}{2 \ln \sigma_g^2}\right) \quad (4)$$

where the log mean diameters is given by

$$\ln(\bar{d}_g) = \frac{1}{L_{Total}} \sum L_i \ln(d_i) \quad (5)$$

and the log standard deviation is given by

$$\ln \sigma_g = \sqrt{\frac{\sum L_i (\ln d_i - \ln \bar{d}_g)^2}{L_{Total}}} \quad (6)$$

To measure fibers from a microscopic image, there are many free software programs available. In our work we use a program called ImageJ (<http://rsb.info.nih.gov/ij/>) to measure length and

diameter of fiber. This program measures the number of pixels in the picture and the scales lengths according to the calibration provided by the user. The picture in Figure 1 shows an example fiber length and fiber diameter from a Scanning Electron Microscope image.

Multiple images may be necessary to obtain a sufficient number of measured fibers. We suggest no fewer than 300 fibers from 10 sample photographs taken at different locations on the fiber mat.

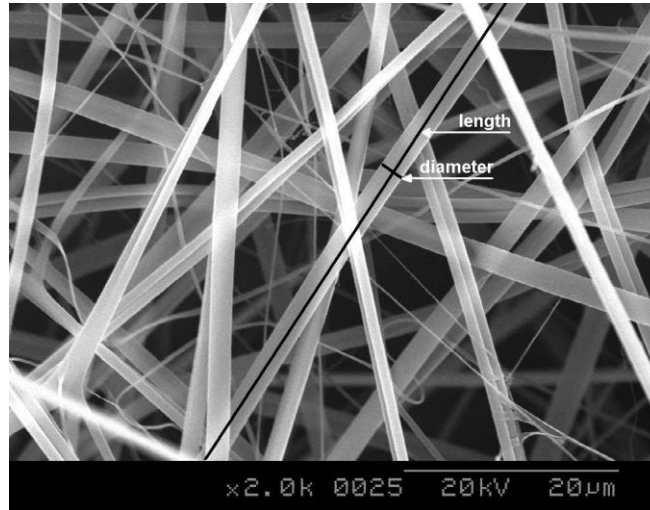


Figure1. SEM image of fiber

The frequency distribution is the slope of the cumulative distributions as given by

$$f_i = \frac{dF_i}{dd_i} \quad (7)$$

As an example, typical fiber diameters and lengths of fibers of those diameters are listed in Table 1. Using Equations (5) and (6) the log mean and standard deviation are calculated to be

$$\bar{d}_g = 156.37 \text{ nm}$$

$$\sigma_g = 1.509$$

An example frequency distribution (plot of f versus fiber diameter, d) is plotted in Figure 2 from data the data in Table 1. The cumulative distribution, F , of a fiber diameter d_i is the sum of the lengths of fibers less than or equal to diameter d_i , as listed in the fourth column of Table 1, divided by the total lengths of all fibers.

Table 1. Typical fiber diameter and length data. The fiber diameters are listed in the column d_i . The total length of fibers of each diameter are in column L_i total. The cumulative amount of fiber lengths are listed in the column Cum L_i and the cumulative frequency is listed in column F .

di, nm	ln(di)	Li total	Cum Li	F
0	0.00	0.00	0.00	0.00
65.8	4.19	14.52	14.52	0.02
83.2	4.42	37.94	52.47	0.08
93.0	4.53	45.60	98.07	0.15
106.1	4.66	12.82	110.89	0.17
117.7	4.77	43.52	154.41	0.24
121.3	4.80	38.11	192.52	0.30
124.8	4.83	6.33	198.84	0.31
131.5	4.88	61.36	260.20	0.41
147.1	4.99	55.56	315.76	0.49
150.0	5.01	30.71	346.46	0.54
158.4	5.07	19.40	365.86	0.57
166.4	5.11	14.43	380.29	0.59
171.5	5.14	57.94	438.23	0.68
176.5	5.17	25.21	463.44	0.72
178.9	5.19	12.29	475.73	0.74
186.0	5.23	8.48	484.21	0.76
212.1	5.36	20.27	504.48	0.79
237.1	5.47	5.69	510.18	0.80
242.5	5.49	6.39	516.57	0.81
249.6	5.52	14.56	531.12	0.83
253.0	5.53	14.51	545.63	0.85
271.2	5.60	12.81	558.44	0.87
277.5	5.63	12.81	571.25	0.89
294.1	5.68	16.29	587.54	0.92
312.7	5.75	14.67	602.20	0.94
328.8	5.80	11.85	614.05	0.96
335.4	5.82	9.30	623.36	0.97
344.3	5.84	2.79	626.15	0.98
394.6	5.98	14.32	640.47	1.00

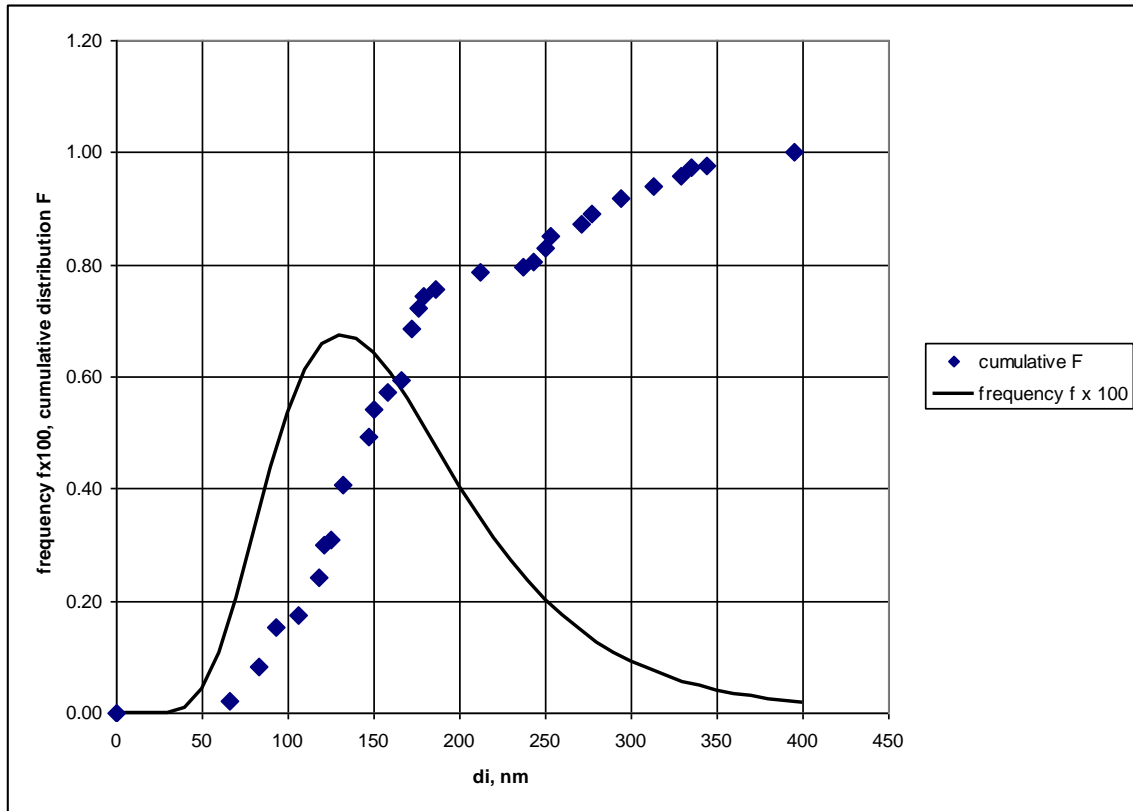


Figure2. Frequency distribution and cumulative frequency distribution

Reference:

- [1] Electrospinning of polymer nanofibers from multiple jets on a porous tubular surface, O O Dosunmu, G G Chase, W Kataphinan and D H Reneker

Keywords

Filtration Media

Fiber

Distribution